Advanced Control of a Quadrotor using Eigenaxis Rotation

Reorientation using LQ-optimal controller with quaternion feedback

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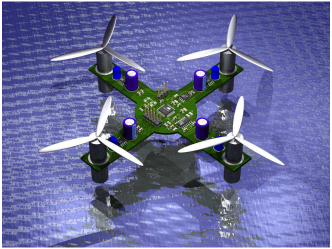
Outline

Practical motivation

UAVs being developped at our institute

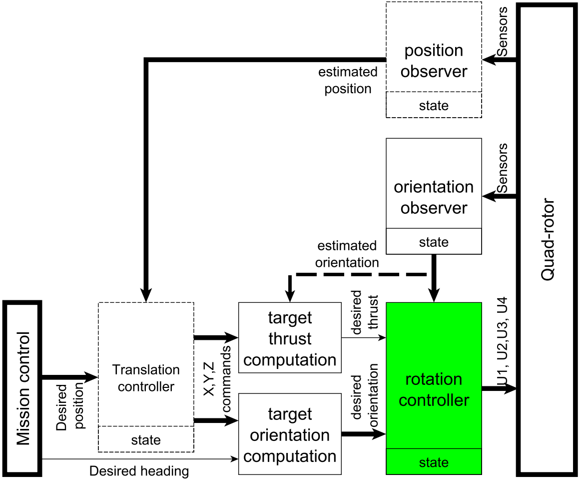


Aiming to synthesize universal rotation (re-orientation) controller

- analyticaly using the precise dynamic model
- with no path constraints or singularities
- energy efficient with fast settling times
- able to handle large maneuvers

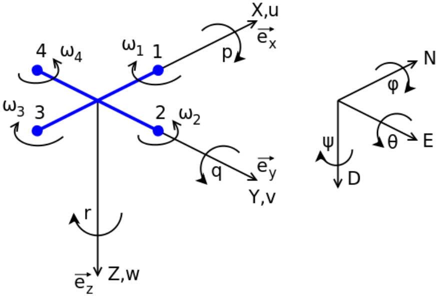
Controller structure

Prioritary dynamics of the rotation over the translation subsystem



Problem definition

Reorientation of a rigid body

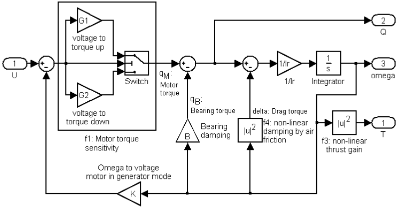


Euler angles vs. Quaternions

Rotation subsystem dynamic model

3DOF rotation subsystem: 9^{th} order non-linear system

- Actuator model



- Rigid body model: acting moments
  - Difference axial thrust and Drag torque from rotors
  - Gyroscopic moments from rotors
  - Aerodynamic resistance moments

Eigenaxis of rotation

Principial axis of re-orientation maneuver from state $\mathbf{Q}$ to $\hat{\mathbf{Q}}$

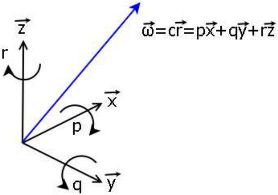
$$\mathbf{E} = \ln(\mathbf{Q}^{-1} \otimes \hat{\mathbf{Q}}) \tag{1}$$

- $\mathbf{E}$ is the eigenaxis vector in body-frame $\mathbf{Q}$.
- To reach the target orientation $\hat{\mathbf{Q}}$, $\mathbf{E}$ must converge to zero.

Quaternion representation offers simple eigenaxis extraction.

Eigenaxis reorientation

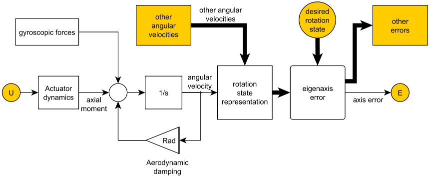
Body rotates around an axis $\mathbf{r}$ iff its angular velocity $\omega = c\mathbf{r}$.



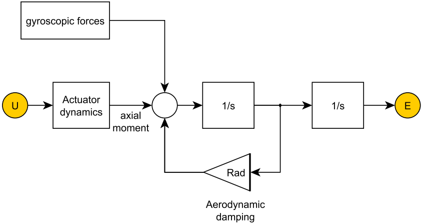
Substituting $\mathbf{r} = \mathbf{E}$, reorientation in time T is performed when $\omega = c\mathbf{E}$, where $\int_0^T c(t)dt = |\mathbf{E}|$. Such reorientation is called *Eigenaxis reorientation*.

Dynamic model with eigenaxis output

The non-linear single-axis decoupled dynamics is shown below



Assuming $\omega = c\mathbf{E}$ during T, model can be linearized



Problem transformation

9^{th} order non-linear system decoupled to three 3^{rd} order linear systems

- Constraint: equal step responses - continuous recalculation
- LQ-optimal controller with Gyroscopic compensator

Advantages over the controller using Euler angles decoupling

- no operating points, no singularities
- velocity corresponds to derivative of eigenaxis error
- spherical interpolation, shortest path

Visualisation

(Loading Circle-m-increase3.mp4)

Controller synthesis

Task: drive the output E of the system

$$\begin{aligned}
\ddot{E}_x &= \frac{l_a}{I_x}\left(T_4 - T_2\right) + \frac{r_{ad\,p}}{I_x}\dot{E}_x + \frac{g_x(\omega, \mathbf{\Omega})}{I_x} \\
\ddot{E}_y &= \frac{l_a}{I_y}\left(T_1 - T_3\right) + \frac{r_{ad\,q}}{I_y}\dot{E}_y + \frac{g_y(\omega, \mathbf{\Omega})}{I_y} \\
\ddot{E}_z &= \frac{1}{I_z}(q_1 - q_2 + q_3 - q_4) + \frac{r_{ad\,r}}{I_z}\dot{E}_z
\end{aligned} \tag{2}$$

where actuator outputs T_j and q_j

$$\begin{aligned}
I_r\dot{\omega}_j &= k\left(\Delta u_j\right) - (m + l)\Delta\omega_j \\
q_j &= I_r\dot{\omega}_j + m\Delta\omega_j + f_4(\omega_0) \\
T_j &= n\Delta\omega_j + f_3(\omega_0)
\end{aligned} \tag{3}$$

to zero using the four inputs Δu_j.

LQ-optimal controller

Model for each axis in matrix form

$$\dot{\mathbf{x}}_j = \begin{bmatrix} -\frac{l+m}{I_r} & 0 & 0 & 0 \\ \frac{nl_a}{I_j} & \frac{r_{adj}}{I_j} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{x}_j + \begin{bmatrix} \frac{k}{I_r} \\ 0 \\ 0 \\ 0 \end{bmatrix} (\tilde{u}_j) \qquad (4)$$

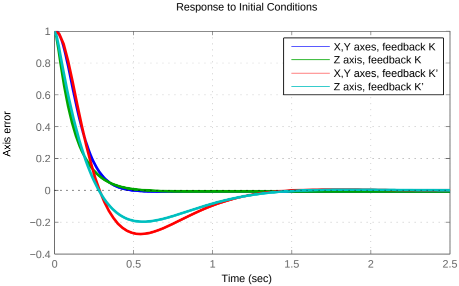
$$E_j = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{x}_j$$

- input is the voltage difference for each axis
- additional integrator for asymptotic tracking
- LQ-optimal controller

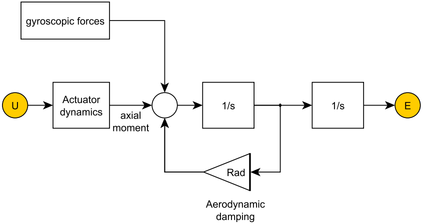
LQ-optimal controller

State variables for feedback

- **E** - computed from $\mathbf{E} = \ln(\mathbf{Q}^{-1} \otimes \hat{\mathbf{Q}})$
- $\int$ **E** - computed using discrete integration
- Ω - measured by gyro
- $\Delta\omega$ - measured by the BLDC drive



Gyroscopic compensation



Gyroscopic forces can be computed from Ω, ω

- Feedforward PD control

Overall control law

Output mapping

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1/2 & 1/4 \\ 1 & 1/2 & 0 & -1/4 \\ 1 & 0 & 1/2 & 1/4 \\ 1 & -1/2 & 0 & -1/4 \end{bmatrix} \begin{bmatrix} {}_T\tilde{u} \\ {}_L\tilde{u}_x + {}_G\tilde{u}_x \\ {}_L\tilde{u}_y + {}_G\tilde{u}_y \\ {}_L\tilde{u}_z \end{bmatrix} \tag{5}$$

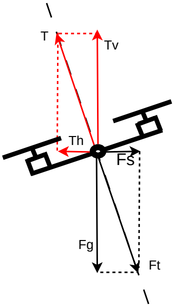
Possible extensions

- overshoot compensation
- inverse non-linearities for the actuator

Interface to precedent layer

Compute target orientation $\hat{\mathbf{Q}}$ from

- desired NED acceleration $\hat{\mathbf{a}}_n$
- desired yaw $\hat{\psi}$



$$\alpha = \arccos \frac{\hat{\mathbf{a}}_n}{|\hat{\mathbf{a}}_n|} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \quad (6)$$

$$\mathbf{Q}_t = \begin{bmatrix} \cos(\frac{\alpha}{2}) \\ \sin(\frac{\alpha}{2}) \frac{\hat{\mathbf{a}}_n}{|\hat{\mathbf{a}}_n|} \times \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \end{bmatrix} \quad (7)$$

$$\mathbf{Q}_y = \begin{bmatrix} \cos(\frac{\hat{\psi}}{2}) & 0 & 0 & \sin(\frac{\hat{\psi}}{2}) \end{bmatrix}^T \quad (8)$$

$$\hat{\mathbf{Q}} = \mathbf{Q}_y \otimes \mathbf{Q}_t \quad (9)$$

Conclusions, discussions, future

- Simple (intuitive) solutions
- Formalized using two concepts: reset observer and modified Smith compensator
- Bias of the rate sensor leads to deterioration but not instability (integration on finite intervals). Bias estimation techniques needed