# Effective control of a quadrotor using eigenaxis rotation

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***Abstract*— These instructions provide basic guidelines for preparing camera-ready (CR) Proceedings-style papers. This document is itself an example of the desired layout for CR papers (inclusive of this abstract). The document contains information regarding desktop publishing format, type sizes, and type faces. Style rules are provided that explain how to handle equations, units, figures, tables, references, abbreviations, and acronyms. Sections are also devoted to the preparation of the references and acknowledgments.**

## I. INTRODUCTION

The key question is how to describe the orientation of rigid body. Altough the quaternions offer a potentially significant advantage, they have been relatively neglecyed in aerospace field. The problem of reorienting a spacecraft in minimum time, often through large angles, without any path constraints, has been addressed in several contexts. A rigid body in any initial orientation ca nbe rotated to a final orientation through single rotation about a fixed axis. Such a rotation, called an eigenaxis rotation, constitutes the "shortest" rotation between the two orientations........

## II. DYNAMIC MODEL

In this paper the quad-rotor dynamics will be treated as a rigid body with six degrees of freedom. Higher order flexible modes will be neglected.

### A. Rigid-Body Dynamics

A simplified schematics of the quadrotor's body-fixed and inertial frames of reference is shown in figure 1. Both of them are right-handed coordinate systems where the right-hand rule applies for determining the direction of a vector cross-product. For the first, $X$,$Y$ and $Z$ are its orthogonal axis with its correspondent body linear velocity vector $\vec{V} = bT\begin{bmatrix} u & v & w \end{bmatrix}$ and angular rate vector $\vec{\Omega} = bT\begin{bmatrix} p & q & r \end{bmatrix}$. The second one is the NED[1] inertial (navigation) reference frame, with which initially the body-fixed coincides (superposition). Moreover, the attitude of the aircraft is assessed by means of rotations around each one of this frame's axis, expressed by terms of the *Euler angles* $\phi$, $\theta$ and $\psi$.

In an inertial frame of reference it is known that the torque (moment) is defined as the time derivative of the angular momentum, $M_{\vec{inertial}} \triangleq \frac{d\vec{L}}{dt} = \frac{d}{dt}\left(I_{\vec{inertial}} \cdot \vec{\Omega}\right)$, where

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[1]NED stands for North-East-Down coordinate system.

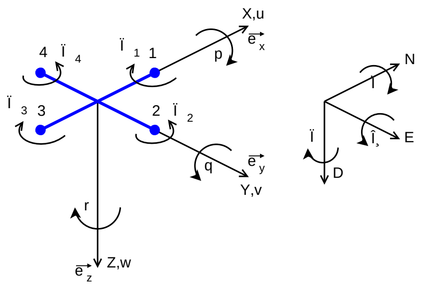


Fig. 1. Quadrotor's fixed-body reference frame.

$I_{\vec{inertial}}$ is the body's inertia tensor measured in inertial reference frame, a 3x3 matrix. However, to simplify the calculations, the $\vec{M}$ shall be rather calculated in the body-fixed rotating frame. In this frame, at least the moment of inertia tensor is constant[2] (and diagonal), $\vec{I} = diag(I_x, I_y, I_z)$.

The angular momentum vector $L$ can now be written as $\vec{L} = \begin{bmatrix} I_x p & I_y q & I_z r \end{bmatrix}^T$. In a rotating reference frame, the time derivative must be replaced with *time derivative in rotating reference frame*, it yields

$$\vec{M} = \left(\frac{d\vec{L}}{dt}\right)_{\text{rot}} + \vec{\Omega} \times \vec{L} \Longleftrightarrow \vec{M} = \vec{I} \cdot \dot{\vec{\Omega}} + \vec{\Omega} \times \left(\vec{I} \cdot \vec{\Omega}\right) \quad (1)$$

which is the particular vector form of Euler's equations. By developing the cross-product term its algebraic form is found as the set of equations

$$\begin{array}{l} M_x = I_x\,\dot{p} + (I_z - I_y)\,q\,r \\ M_y = I_y\,\dot{q} + (I_x - I_z)\,r\,p \\ M_z = I_z\,\dot{r} + (I_y - I_x)\,p\,q \end{array} \quad (2)$$

which are also referred to as the Euler *moment equations*. One can note the physical natural sense in these equations: the simultaneous rotation around two axis will generate a torque around a third axis, given that the previous causal two axis don't have the same inertia.

Similarly to the reasoning applied until here to the rotational aspect of the rigid-body dynamics, in the translational case a force is generated, according to Newton's 2$^{\text{nd}}$ law, as $\vec{F} \triangleq \frac{d\vec{P}}{dt} = \frac{d}{dt}\left(m \cdot \vec{V}\right)$, where $m$ is the total mass of the quadrotor in whose center the origin of the aircraft's fixed-body rotating frame is located. Once again turning to the body-fixed rotational frame and defining linear momentum $\vec{P} = \begin{bmatrix} m\,u & m\,v & m\,w \end{bmatrix}^T$, the relation is expressed as

[2]View of a quad-rotor as rigid body with constant mass

$$\vec{F} = \left(\frac{d\vec{P}}{dt}\right)_{\text{rot}} + \vec{\Omega} \times \vec{P} \Longleftrightarrow \vec{F} = m\left(\dot{\vec{V}} + \vec{\Omega} \times \vec{V}\right) \quad (3)$$

By solving the cross-product and once more using the *p,q,r* aerospace notation of Euler angular rates, the set of *force equations* is obtained

$$\begin{aligned} F_x &= m\,(\dot{u} + q\,w - v\,r) \\ F_y &= m\,(\dot{v} + r\,u - w\,p) \\ F_z &= m\,(\dot{w} + p\,v - q\,u) \end{aligned} \quad (4)$$

### B. Actuator dynamics

In this section, a simplified model of a BLDC (brushless DC) motor with attached propeller will be introduced. Input to the actuator is applied voltage $U_j$ and output vector is formed of propeller angular velocity $\omega_j$, axial thrust $T_j$ and torque $Q_j$. To simplify the model, stress-free motor will be treated as ordinary, linear first-order system. The fast dynamics caused by magnetic inductance will be neglected. Nevertheless, experiments have shown that a conventional cheap BLDC motor with controller has different dynamics when accelerating and deccelerating due the PWM driving strategy. We have developped a special motor controller in order to minimize this effect, however there is still certain difference between the two poles, described by $f_1(u_j - K\omega_j)$ in the model.

The propeller adds non-linear damping $\delta_j = f_4(\omega_j)$ to the motor, caused by the air friction of the blades. It can be found in the literature that drag torque depends polynomially on $\omega$. This damping makes non-linear not only the steady-state gain, but aswell the entire dynamics, which is often mistreated. Following the $2^{nd}$ Newton's law $\dot{\omega}_j = (T_M - T_B - \delta_j)/I_r$. Since the torque can transfer from the propeller to the body through motor only (air is independent liquid medium with its own inertia), the output torque equals $Q = T_M - T_B$ which is equal to $\delta_j$ in steady state.

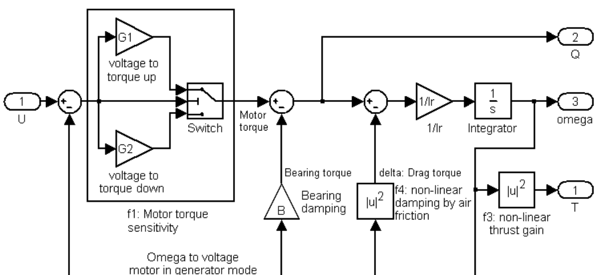


Fig. 2. actuator model

The model dynamic is described as follows

$$I_r\dot{\omega}_j = f_1(u_j - K\omega_j) - B\omega_j - f_4(\omega_j) \quad (5)$$

and output relations

$$\begin{aligned} Q_j &= f_1(u_j - K\omega_j) - B\omega_j = I_r\dot{\omega}_j + \delta_j \\ T_j &= f_3(\omega_j) \end{aligned} \quad (6)$$

To linearize the dynamics, in (5) the non-linear sensitivity function, caused by the assymetric up and down step response can be meaned, $f_1(u_j - K\omega_j) = \frac{G_1+G_2}{2}(u_j - K\omega_j)$. Polynomial relations $f_3(\omega_j)$ and $f_4(\omega_j)$ can be linearized around operating point $\omega_0$, where the sum of all four propeller thrusts equals quadrotor mass, $f_3(\omega_j) \equiv \beta\Delta\omega_j$ and $\delta_j = f_4(\omega_j) \equiv \alpha\Delta\omega_j$, leading to linearized dynamics describing the actuator as ordinary first order system

$$I_r\dot{\omega}_j = ku_j - l\omega_j - m\Delta\omega_j \quad (7)$$

and linearized output relations

$$\begin{aligned} Q_j &= I_r\dot{\omega}_j + m\Delta\omega_j \\ T_j &= n\Delta\omega_j \end{aligned} \quad (8)$$

where $k = \frac{G_1+G_2}{2}$, $l = K\frac{G_1+G_2}{2}$, $m = \dot{f}_4(\omega_0)$ and $n = \dot{f}_3(\omega_0)$. Note that in linear approximation around $\omega_0$, drag trust $\delta_j$ is proportional to axial thrust $T_j$, namely $\delta_j = cT_j$, with $c = \frac{\alpha}{\beta}$.

### C. Acting Forces and Moments

Hereby all those forces and moments which act on the model derived in the previous section shall be addressed. Assuming low translational speeds of the quadrotor, the aerodynamic forces exerted upon it are very small and will thus be disregarded.

*1) Gyroscopic Moments from Rotors:* The four rotors induce gyroscopic moments on the aircraft due to their angular velocity, $\omega_j$. This is one of the two main mechanisms to cause the quadrotor to yaw. Moreover, their gyroscopic effect affects also, although relatively less intense, the other two axes. This will also be modelled herein.

Considering the spin direction of rotor $j = 1$, the gyroscopic torque resulting from the interaction of the rotor with the rotating aircraft and acted on the generic rotor $j = 1 \ldots 4$ is given, similarly to equation 1, by

$$\vec{M^j} = \frac{d\vec{L^j}}{dt} + \vec{\Omega} \times \vec{L^j} = \vec{I^j} \cdot \dot{\vec{\omega^j}} + \vec{\Omega} \times \left(\vec{I^j} \cdot \vec{\omega^j}\right) \quad (9)$$

where $\vec{I^j}$ is the gyroscopic inertia, namely that of the rotating part of the rotor. However, the direction of $\vec{\omega^j}$ coincides with the $Z$-axis of the aircraft whereas all its other components are zero, aswell as for $\vec{I^j}$, therefore equation 9 is simplified and the set of 3 algebraic equations expressing the gyroscopic torques acted on a rotor $j = 1 \ldots 4$ is

$$\begin{aligned} M_x^j &= I_j\,\omega_j\,q \\ M_y^j &= -I_j\,\omega_j\,p \\ M_z^j &= I_j\,\dot{\omega}_j \end{aligned} \quad (10)$$

Now for taking into consideration all rotors, all angular speeds need to be summed up according to their respective sign (direction), namely as

$$\omega_R = \sum_{j=1}^{4} \omega_j = \omega_1 - \omega_2 + \omega_3 - \omega_4 \tag{11}$$

Assuming that the gyroscopic masses of each rotor are the same, $I_R$, and finally considering the reaction torques on the aircraft's body, hence inverting the sign of the set of equations 10, the total rotor gyroscopic moment exerted on the aircraft's body is

$$\begin{aligned} M_x^R &= -I_R\,\omega_R\,q = -L_R\,q \\ M_y^R &= I_R\,\omega_R\,p = L_R\,p \\ M_z^R &= -I_R\,\dot{\omega_R} = -\dot{L_R} \end{aligned} \tag{12}$$

*2) Earth's Gravity:* The Earth's gravitational field around the quadrotor causes its weight force to act upon it on its mass center. This is modeled in the inertial (navigation) frame again simply by Newton's 2nd law as

$$n\vec{g}F = \begin{bmatrix} 0 & 0 & m\,g \end{bmatrix}^T \tag{13}$$

where $g = 9,81\,m/s^2$ is the absolute value of Earth's gravity acceleration. However this acting force needs to be considered in the body-fixed frame, therefore the need for a rotation matrix appears........

*3) Thrust and torque:* The effects of the thrust generated by the rotors through their attached propellers is directly calculated in the body-fixed frame.

First of all, the actuators generates a thrust used to maintain the aircraft in the air. This force is always aligned with the body-fixed $Z$-axis, thus

$$b\vec{T}F = \begin{bmatrix} 0 & 0 & -T \end{bmatrix}^T \tag{14}$$

where $T = \sum_{j=1}^{4} T_i$ is the sum of the positive thrust forces produced by each rotor.

The difference in thrust produced by the propellers in the same axis define a moment around that axis. Also, the drag thrust reaction $\delta_j$ caused by the air friction of the blades (the second mechanism that causes the quad-rotor to yaw), hence

$$\begin{aligned} M_x^T &= l_a\,(T_4 - T_2) \\ M_y^T &= l_a\,(T_1 - T_3) \\ M_z^T &= \delta_1 - \delta_2 + \delta_3 - \delta_4 \end{aligned} \tag{15}$$

where $l_a$ is the lever length of each of the quadrotor's arms, assumed to be the same for all of them.

As shown in section *Propeller dynamics* that drag thrust $\delta_j$ can be treated as linearly dependent to $T_j$. Accepting such presumption all thrusts can be mapped to moments through a matrix

$$\begin{bmatrix} T \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -l_a & 0 & l_a \\ -l_a & 0 & l_a & 0 \\ c & -c & c & -c \end{bmatrix} \times \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} \tag{16}$$

Where c is a scalar such that $\delta_j = c\,T_j$. The matrix is invertible, thus the moments are uniquely determinig the applied thrusts.

### D. Non-Linear Model

Being already assessed the forces and moments acting on the quadrotor, its non-linear model is obtained by applying the set of equations 12 and 15 into the left side of 2. After rearranging the terms and isolating the angular accelerations, the *moment equations* are obtained

$$\begin{cases} \dot{p} = \dfrac{l_a}{I_x}\,(T_4 - T_2) + \dfrac{(I_y - I_z)}{I_x} q\,r - \dfrac{L_R\,q}{I_x} \\ \dot{q} = \dfrac{l_a}{I_y}\,(T_1 - T_3) - \dfrac{(I_x - I_z)}{I_y} r\,p + \dfrac{L_R\,p}{I_y} \\ \dot{r} = \dfrac{1}{I_z}(\delta_1 - \delta_2 + \delta_3 - \delta_4) - \dfrac{\dot{L_R}}{I_z} \end{cases} \tag{17}$$

whereas by inserting the set of equations **??** into the left side of 4 and isolating the translational accelerations, the *force equations* arise

$$\begin{cases} \dot{u} = -q\,w + v\,r - g_B \\ \dot{v} = -r\,u + w\,p + g_B \\ \dot{w} = -p\,v + q\,u + g_B - \dfrac{T}{m} \end{cases} \tag{18}$$

## III. Controller design

### A. Control goals

In addition of the angular rate from each of the propellers, there are totaly 10 outputs, $\vec{Y} = \begin{bmatrix} \phi & \theta & \psi & x & y & z & \omega_1 & \omega_2 & \omega_3 & \omega_4 \end{bmatrix}$ (disregarding their time derivations and integrals), that can be naturally measured or estimated from physical sensor readings and 4 inputs, $\vec{U} = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}$, representing the voltage (PWM duty cycle) level for each of the actuators.

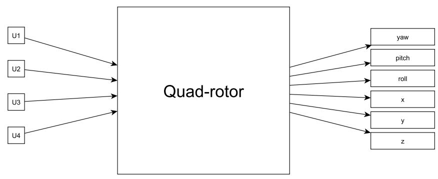


Fig. 3. basic model inputs/outputs

Target is a space orientation and/or space position control. As the model has less inputs than outputs, not all of them can be controlled simultaneously ,i.e. the device is not holonomic. In gravity-free environment, the non-holonomic idea can be brought to all parts: the device can only move, or generate thrust $T$ along Z-axis (2-wheel robot analogy). In a NED frame, the generated thrust vector $\vec{T_{NED}}$ can be separated into horizontal and vertical parts such as $\vec{T_{NED}} = \begin{bmatrix} \vec{T_{NEDh}} & T_{NEDv} \end{bmatrix}$. Horizontal part $\vec{T_{NEDh}}$ is a vector that accelrates the aircraft in horizontal (North-East) plane and the vertical part $T_{NEDv}$ is a scalar that must be equal to Earth's gravity magnitude in order to maintain the altitude. However, direction of $\vec{T_{NEDh}}$ depends directly on $\phi$, and $\theta$, accepting the Euler angles as orientation representation, meaning that theese outputs can not be controlled independently. Since the position is prior to orientation and most likely what is to be controlled, the target control vector is reduced to: $\vec{Y_{control}} = \begin{bmatrix} x & y & z & \psi \end{bmatrix}$

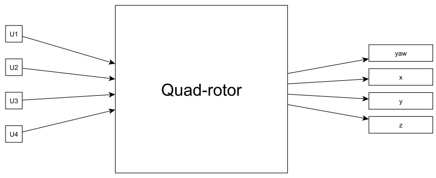


Fig. 4. basic model inputs/outputs

Respecting that above relationship between the attitude and the horizontal acceleration, the following control structure is proposed for the overall system

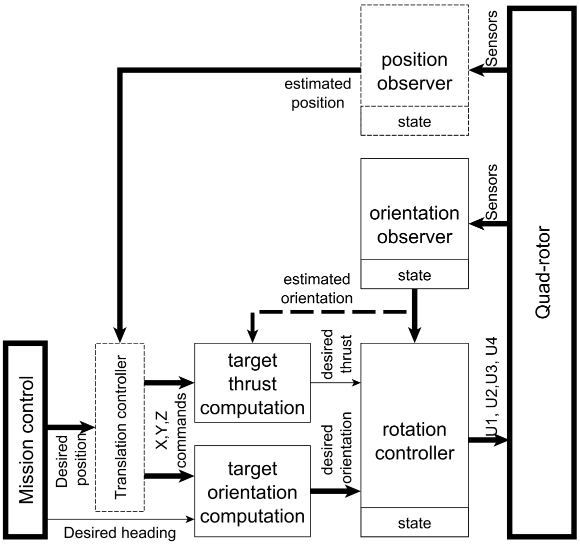


Fig. 5. basic model inputs/outputs

## B. Rotation representation

There are several methods how to describe a spatial rotation. The most common method is the Euler angles, $\phi, \theta, \psi$. In avionic, these angles are used directly as proportional error inputs to the controllers. However, $\phi, \theta, \psi$ represents consecutive rotations applied to the rigid body. Thus, the controllers should act in the same order, stabilising the axes respectively. In a rotational state far from the operating point during simultaneous stabilisation of all the axes based on euler angles error, the highly non-linear, even singular, errors are met as the angle change does not correspond to angular velocity.

Each spatial rotation can be expressed using the eigenaxis vector and rotation angle. Rotation quaternions are effectively used for such expression. Nevertheless, they add certain redundancy (4 scalar numbers to describe 3 degrees of freedom) in opposite to euler angles. Rotation quaternions have unitary norm and the eigenaxis $r$ is a unit vector also. A special non-cummutative multiplication operation is defined for the quaternions, denoted with the $\times$ operator. Multiplying two or more rotation quaternions produces another roatation quaternion that represents the two consecutive rotations performed on the coordinate system. The non-commutativity of $\times$ operator reflects the fact that a rigid body is in different orientation state after a two consecutive rotations, depending on their order. A conjugate quaternion $Q^{-1}$ represents an inverse rotation. Relations between the quaternions, angle $\alpha$ of rotation and eigenaxis vector $\vec{r}$ are defined as follows

$$Q = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \cos(\frac{\alpha}{2}) \\ sin(\frac{\alpha}{2})\vec{r} \end{bmatrix} \tag{19}$$

$$\ln q = \frac{\vec{r}\alpha}{2} \tag{20}$$

## C. Desired rotation and thrust

Respecting the control structure proposed in 5, following the supreme dynamics of rotation subsystem over the translation subsystem, is needed first to transform the target horizontal acceleration and yaw command into target absolute rotation $Q_d$ and thrust $T$ in body frame. For tilt, we first introduce a vector $\vec{v} = \begin{bmatrix} x & y & g+z \end{bmatrix}^T$ representing target linear acceleration in NED frame, and then compute angle of tilt rotation $\alpha$ as angle between $v$ and unitary vector pointing up

$$\alpha = \arccos \frac{\vec{v}}{|\vec{v}|} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \tag{21}$$

$g$ is magnitude of Earth's acceleration for preserving the real units. The resulting tilt quaternion $Q_t$ is then formed as rotation with eigenaxis composed of vector cross product

$$Q_t = \begin{bmatrix} \cos(\frac{\alpha}{2}) \\ sin(\frac{\alpha}{2})\frac{\vec{v}}{|\vec{v}|} \times \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \end{bmatrix} \tag{22}$$

Then, for yaw rotation simply define a rotation around down-axis in NED-frame

$$Q_y = \begin{bmatrix} \cos(\frac{\psi}{2}) & 0 & 0 & sin(\frac{\psi}{2}) \end{bmatrix}^T \tag{23}$$

Finally, $Q_d$ can be composed by multiplying theese two quaternions. Note that this step can be done in both orderings. First presented in (26) applies the tilt first preserving the absolute accelerations in NED frame whereas the second one will apply the heading first, so the acceleration commands will be relative to heading and device behaves just like manned helicopter in view of translation controller.

$$\begin{gathered} Q_d = Q_y \times Q_t \\ Q_d' = Q_t \times Q_y \end{gathered} \tag{24}$$

The desired sum of propeller thrusts magnitude in body frame is then simply defined as

$$T_d = |\vec{v}| \tag{25}$$

Another way to compute $T_d$ in order to maintain the target thrust in Z-axis of NED frame during entire correction maneuver could be done by dynamically mapping the desired vertical thrust to body frame

$$T_d' = \frac{g+z}{cos(\beta)} \tag{26}$$

where $\beta$ is actual angle between $\begin{bmatrix} 0 & 0 & -1 \end{bmatrix}^T$ and unitary vector pointing down in body-frame.

### D. Control of rotation using eigenaxis

Assuming we have the orientation estimation subsystem onboard the flying device which outputs the actual absolute orientation state based on the sensor readings, we can define the error quaternion $Q_e$ such as

$$Q_e = Q^{-1} \times Q_d \tag{27}$$

where $Q$ is current orientation state representing quaternion and $Q_d$ is our target state. $Q_e$ is left-invariant in view of $Q$ and stands for a rotation needed to perform to reach state $Q_d$ from state $Q$, namely $Q_d = Q \times Q_e$. Both $Q_e$ and $Q_e^{-1}$ leads to target state, since $Q_d$ represents same state as $Q_d^{-1}$. Such, it is often neede to select $Q_e = min(Q_e, Q_e^{-1})$. Based on the knowledge of rigid body angular velocities, a orientation quaternion propagation through time is defined as

$$\dot{Q} = W(\vec{\omega})Q = \frac{1}{2}\begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & -r & q \\ q & r & 0 & -p \\ r & -q & p & 0 \end{bmatrix} Q \tag{28}$$

The state $Q$ must be continuously renormalized in order to preserve the unitary norm. Time propagation of $Q$ can be also rewritten as multiplication by another quaternion formed out of the angular velocities, $Q_\omega = \begin{bmatrix} 0 & p & q & r \end{bmatrix}^T$, such as the time derivation of the elements responds to infinetezimaly small rotations of $Q$ by $Q_\omega$

$$\dot{Q} = \frac{1}{2} Q \times \begin{bmatrix} 0 & p & q & r \end{bmatrix}^T \tag{29}$$

Assuming the $\vec{\omega} = c\hat{\vec{\omega}}$ where $\hat{\vec{\omega}}$ is unitary and constant over the time period $T$, integral of (29) starting from the quaternion $Q_0$ that propagates through time, after $T$ period the state $Q_d$ can be defined as

$$Q_d = e^{W(\hat{\vec{\omega}})I} Q = Q \times Q_r, \; I = \int_0^T c(\tau)d\tau \tag{30}$$

where

$$Q_r = \begin{bmatrix} a & b\hat{\vec{\omega}}I \end{bmatrix}^T, \; I <= \pi \tag{31}$$

Where $a$ and $b$ are scalars used to preserve the unitary norm, $|Q_r| = 1$ and $a = cos(\frac{\alpha}{2})$. Since the total angle of rotation must be $\alpha = I$, respecting the unitary $\hat{\vec{\omega}}$, substituting $Q_r$ to $Q_e$ in (27), and then comparing with (23), (20) leads to

$$\vec{r}\alpha = 2\ln(Q \times Q_d^{-1}) = \hat{\vec{\omega}} \int_0^T c(\tau)d\tau \tag{32}$$

meaning that correction of the error rotation can be achieved by driving the $\vec{\omega}$ proportionally to $\vec{r}$, $\vec{\omega} = c\vec{r}$ with constraint $I = \alpha$, wich is satisfied by each controller-plant system that uses $\vec{r}\alpha$ as error input, shows equal dynamics for all axes disregarding the gyrosopic effects and fullfills asymptotic tracking.

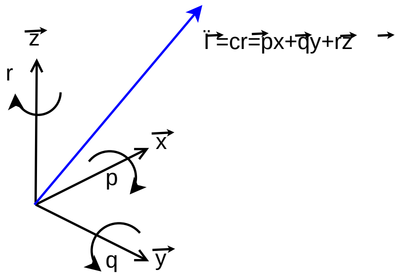


Fig. 6. Releation of eigenaxis to body angular velocities

Similar proof could be done for eigenaxis of $R$ rotating matrix. Moreover, the $\vec{r}\alpha$ vector can be used as feedback in decoupled axial controllers instead of $\phi, \theta, \psi$ with significantly less errors, as the poportional term is always inegral of derivative term (angular velocity) [], the eigenaxis rotation represents the 'shortest' possible correction maneuver (quaternion interpolation) []. The only restriction is the constant $\hat{\vec{\omega}}$ during correction maneuver, requiring the equaly adjusted dynamics for all axes. Nevertheless, if the feedback controller recalculates the eigenaxis error each time step, the errors are often more acceptable during large maneuvers.

### E. Axial decoupling and controller synthesis

Using the superposition principle on acting thrust and drag torque on each axis in linearized actuator model from (7), (8), the separate decoupled axial dynamics is shown in fig. 7.

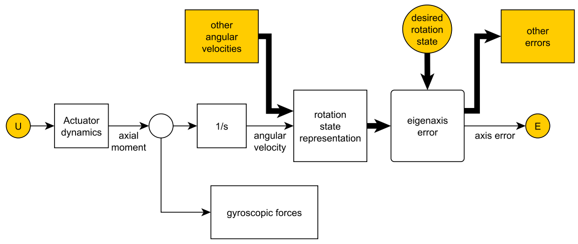


Fig. 7. Single axis dynamic model

During the correction maneuver, assuming the equal step responses for all axes from (32) the eigenaxis error can be

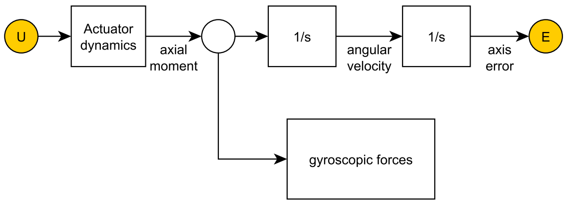


Fig. 8. Single axis dynamic model during correction maneuver

treated as simple integral of angular velocity. The diagram shown in fig. 7 reduces to fig. 8.

$$\begin{aligned} \dddot{E}_x &= \frac{l_a}{I_x}(T_4 - T_2) + g_x(\vec{\omega}, \vec{\Omega}) \\ \dddot{E}_y &= \frac{l_a}{I_y}(T_1 - T_3) + g_y(\vec{\omega}, \vec{\Omega}) \\ \dddot{E}_z &= \frac{1}{I_z}(Q_1 - Q_2 + Q_3 - Q_4) + g_z(\vec{\omega}, \vec{\Omega}) \end{aligned} \tag{33}$$

Hereby the conventional LQR/LQI control strategy will be presented for decoupled axial dynamics in combination with feed-forward control to compensate the gyroscopic torque. Decoupled dynamics for error $X$ and $Y$ axes can be written in matrix-form, composed of (7), (8), (33)

??? - linearizace?

$$\dot{x}_j = \begin{bmatrix} -\frac{l+m}{I_r} & 0 & 0 & 0 \\ n & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x_j + \begin{bmatrix} \frac{k}{I_r} \\ 0 \\ 0 \\ 0 \end{bmatrix} (U_j) \tag{34}$$

$$E_j = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} x_j$$

where $j$ is the axis index, $j = (x, y)$, $U_x = (u_4 - u_2)$ and $U_y = (u_1 - u_3)$. Similarly for the vertical axis

$$\dot{x}_z = \begin{bmatrix} -\frac{l+m}{I_r} & 0 & 0 & 0 \\ -l & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x_z + \begin{bmatrix} \frac{k}{I_r} \\ k \\ 0 \\ 0 \end{bmatrix} (U_z) \tag{35}$$

$$E_z = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} x_z$$

where $U_z = (u_1 - u_2 + u_3 - u_4)$.

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### *F. Full-Size Camera-Ready (CR) Copy*

If you have desktop publishing facilities, (the use of a computer to aid in the assembly of words and illustrations on pages) prepare your CR paper in full-size format, on paper 21.6 x 27.9 cm (8.5 x 11 in or 51 x 66 picas). It must be output on a printer (e.g., laser printer) having 300 dots/in, or better, resolution. Lesser quality printers, such as dot matrix printers, are not acceptable, as the manuscript will not reproduce the desired quality.

*1) Typefaces and Sizes::* There are many different typefaces and a large variety of fonts (a complete set of characters in the same typeface, style, and size). Please use a proportional serif typeface such as Times Roman, or Dutch. If these are not available to you, use the closest typeface you can. The minimum typesize for the body of the text is 10 point. The minimum size for applications like table captions, footnotes, and text subscripts is 8 point. As an aid in gauging type size, 1 point is about 0.35 mm (1/72in). Examples are as follows:

*2) Format::* In formatting your original 8.5" x 11" page, set top and bottom margins to 25 mm (1 in or 6 picas), and left and right margins to about 18 mm (0.7 in or 4 picas). The column width is 88 mm (3.5 in or 21 picas). The space between the two columns is 5 mm(0.2 in or 1 pica). Paragraph indentation is about 3.5 mm (0.14 in or 1 pica). Left- and right-justify your columns. Cut A4 papers to 28 cm. Use either one or two spaces between sections, and between text and tables or figures, to adjust the column length. On the last page of your paper, try to adjust the lengths of the two-columns so that they are the same. Use automatic hyphenation and check spelling. Either digitize or paste your figures.

TABLE I
AN EXAMPLE OF A TABLE

| | |
|---|---|
| One | Two |
| Three | Four |

## IV. UNITS

Metric units are preferred for use in IEEE publications in light of their international readership and the inherent convenience of these units in many fields. In particular, the use of the International System of Units (SI Units) is advocated. This system includes a subsystem the MKSA units, which are based on the meter, kilogram, second, and ampere. British units may be used as secondary units (in parenthesis). An exception is when British units are used as identifiers in trade, such as, 3.5 inch disk drive.

## V. ADDITIONAL REQUIREMENTS

### *A. Figures and Tables*

Position figures and tables at the tops and bottoms of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table captions should be above the tables. Avoid placing figures and tables before their first mention in the text. Use the abbreviation "Fig. 1", even at the beginning of a sentence. Figure axis labels are often a source of confusion. Try to use words rather then symbols. As an example write the quantity "Inductance", or "Inductance L", not just. Put units in parentheses. Do not label axes only with units. In the example, write

"Inductance (mH)", or "Inductance L (mH)", not just "mH". Do not label axes with the ratio of quantities and units. For example, write "Temperature (K)", not "Temperature/K".

### B. Numbering

Number reference citations consecutively in square brackets [1]. The sentence punctuation follows the brackets [2]. Refer simply to the reference number, as in [3]. Do not use "ref. [3]" or "reference [3]". Number footnotes separately in superscripts[3] Place the actual footnote at the bottom of the column in which it is cited. Do not put footnotes in the reference list. Use letters for table footnotes (see Table I).

### C. Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, CGS, ac, dc, and rms do not have to be defined. Do not use abbreviations in the title unless they are unavoidable.

### D. Equations

Number equations consecutively with equation numbers in parentheses flush with the right margin, as in (1). To make your equations more compact you may use the solidus (/), the exp. function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather then hyphen for a minus sign. Use parentheses to avoid ambiguities in the denominator. Punctuate equations with commas or periods when they are part of a sentence:

$$\Gamma_2 a^2 + \Gamma_3 a^3 + \Gamma_4 a^4 + ... = \lambda \Lambda(x),$$

where $\lambda$ is an auxiliary parameter.

Be sure that the symbols in your equation have been defined before the equation appears or immediately following. Use "(1)," not "Eq. (1)" or "Equation (1)," except at the beginning of a sentence: "Equation (1) is ...".

Fig. 9. Inductance of oscillation winding on amorphous magnetic core versus DC bias magnetic field

## VI. CONCLUSIONS AND FUTURE WORKS

### A. Conclusions

This is a repeat. Position figures and tables at the tops and bottoms of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table captions should be above the tables. Avoid placing figures and tables before their first mention in the text. Use the abbreviation "Fig. 1", even at the beginning of a sentence. Figure axis labels are often a source of confusion. Try to use words rather then symbols. As an example write the quantity "Inductance", or "Inductance L", not just. Put units in parentheses. Do not label axes only with units. In the example, write "Inductance (mH)", or "Inductance L (mH)", not just "mH". Do not label axes with the ratio of quantities and units. For example, write "Temperature (K)", not "Temperature/K".

[3]This is a footnote

### B. Future Works

This is a repeat. Position figures and tables at the tops and bottoms of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table captions should be above the tables. Avoid placing figures and tables before their first mention in the text. Use the abbreviation "Fig. 1", even at the beginning of a sentence. Figure axis labels are often a source of confusion. Try to use words rather then symbols. As an example write the quantity "Inductance", or "Inductance L", not just. Put units in parentheses. Do not label axes only with units. In the example, write "Inductance (mH)", or "Inductance L (mH)", not just "mH". Do not label axes with the ratio of quantities and units. For example, write "Temperature (K)", not "Temperature/K".

## VII. ACKNOWLEDGMENTS

The authors gratefully acknowledge the contribution of National Research Organization and reviewers' comments.

References are important to the reader; therefore, each citation must be complete and correct. If at all possible, references should be commonly available publications.

## REFERENCES

[1] J.G.F. Francis, The QR Transformation I, *Comput. J.*, vol. 4, 1961, pp 265-271.

[2] H. Kwakernaak and R. Sivan, *Modern Signals and Systems*, Prentice Hall, Englewood Cliffs, NJ; 1991.

[3] D. Boley and R. Maier, "A Parallel QR Algorithm for the Non-Symmetric Eigenvalue Algorithm", *in Third SIAM Conference on Applied Linear Algebra*, Madison, WI, 1988, pp. A20.